Exploring the Historical Debates on Irrational Numbers using Neutrosophic Logic as a Balance between Intuition and Rational

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Abstract

A short book by Dennis P. Allen, Jr, a senior mathematician, inspires this article, and henceforth it is dedicated to him. A good movie about S. Ramanujan, *The Man who knew Infinity,* also triggers this work. As a note, first of all, this is not a conventional math paper. Instead, its purpose is to dig deeper into how a mathematician or a scientist should deal with intuition and balance it with a logical thinking process. Literature exploration on important inventions in mathematics becomes the method of this study combined with analysis of Iain McGilchrist's theory and Wittgenstein's Philosophy of Language added with the Cognitive Language Theory. The findings show the absolutistic view of rationality or rational number will not suffice to give a holistic insight into reality. Such finding serves as a reminder concerning whom should be the Master and who should be the emissary in the path toward knowledge. In the end of this paper, we argue based on Neutrosophic Logic that we need to consider "intuilytics" which combines both parts of brain hemispheres.

Keywords: Irrational numbers, intuition, mathematics, right-left brain, Logico Filosophico, Cognitive Linguistics Analysis, Neutrosophic Logic, intuilytics

Introduction

Although this is not a movie review, allow us to cite a review of a movie on S. Ramanujan. Krishnaswami Alladi wrote about movie: *The Man who knew Infinity*, which depicts a story on how a great mathematician from India met with another great mathematician in Cambridge, Prof G. Hardy.[1]

While that movie is an exciting introduction to Ramanujan's remarkable invention of partition theorem, and also the number 1729 (discovery inspired by a taxi-cab number in London), what seems intriguing is that the movie sharpens the contrasts between two significant figures in mathematics at their time. First is G. Hardy, who used a rigorous math-proving method, while the second, Ramanujan was intuitive.

[1] Krishnaswami Alladi, REVIEW OF THE MOVIE ON THE MATHEMATICAL GENIUS RAMANUJAN. (unknown date)

While one got their own hypothesis on how things should work on discovering new science and mathematics ideas from G. Hardy's famous book: *A Mathematician's Apology*, along with a more recent book by a psychiatrist Iain McGilchrist, something fresh might emerge.

Hardy's account on Hippasus story

A book was written by mathematician Dennis Allen, Jr, as a memoir of his long career in various diverse areas in science serves as this article point of departure. Allen opens Chapter One of his book by quoting Thomas Phipp, Jr.'s remark on G. Hardy's book *A Mathematician's Apology* :

> "People like G.H. Hardy ('*A Mathematician's Apology*, Cambridge, 1969), who forms the chief role models for modern pure mathematicians, have charted just this regrettable course — with a cost to mathematics that can never be reckoned. Hardy incidentally uses the word 'significance' where I use 'fruitfulness'. His 'mathematician's apology' consists of dividing mathematics into two disjoint halves, one 'trivial' or 'useful' that he consigns to perdition, the other 'real', useless, and …on both aesthetic and moral grounds. Writing in 1940, he says that 'No one has yet discovered any warlike purpose to be served by the theory of numbers or relativity', and by such reasoning places ….subjects on the moral plane of the angels along with all 'real' mathematicians."

With those statements, such as the usefulness and real, beauty mathematics which serve for nothing, the 26-dimensional bosonic superstring theories or something to serve people in doing better to improve their life apparently, it is not just a problem of fancy mathematics is at stake. However, those judgmental statements need deeper analysis as it brings forward absolute rationalism.

Succinctly, this article posits the following question: which is real mathematics? It is "something with all glory and fanciness," or those which is "closer to realism?" If one call "realism" helpful in doing mathematics, does it mean that intuition in developing new ideas can play roles in the equations? Then, the main question is whether logical

processes are the only method that humans should rely on or another possibility co-exists. The hypothesis is that the abolutistic logical or rational approach is insufficient to depict reality as it needs an intuitive approach to yield a holistic result together.

The method of this explorative study is literature exploration. Thus it belongs to a qualitative methodology. This short article's foci are as follows: first of all, the discussion will be on the classic story of Hippasus' invention: irrational numbers versus the famous Pythagoreans' approach. Then, the exploration of McGilchrist's concept of the Right and Left brain will follow. The last is the analysis on Logico Filosofico of Wittgenstein and Lakoff's *Cognitive Linguistic Theory* to shed light on the issues.

Literature Analysis

What happened between Hippasus and Pythagoreans rationalism

In discussing G. Hardy's discovery of irrational numbers, Allen continues:

> "Further, Hardy's philosophy as set forth in his above mentioned book is fanciful in other ways too, as for example in his (with Wright) "*An Introduction to the Theory of Numbers*" (fourth edition) on page 39, he ascribes the proof that the square root of two is irrational—this being the first irrational number to be discovered — to Pythagoras."

Peter Gainsford also wrote :

> "There is a widespread notion that the discovery of irrational numbers was a thing of horror to the ancient Greeks, especially for the school of Pythagoras. Pythagoras is best known today for a famous theorem about right-angled triangles, but in antiquity, his significant contribution lies in the fact that he was a semi-legendary guru who founded a philosophical-religious sect in southern Italy. No writings by Pythagoras himself survive (and it is unlikely he ever wrote any). The oral records about the sect sound bizarre at times such as the Pythagoreans conveyed their teachings only in a cave or.they had weirdly specific beliefs about reincarnation, and they venerated unexpected plants like fava beans and mallow. The vast majority of this information is reported very late and is almost certainly false; the bits that are true (whichever ones they are) are difficult to understand out of context."

Gainsford went on with a quote from Kleine's book, discussing Hippasus:

> "In 1972, the mathematician Morris Kline wrote in his book *Mathematical Thought from Ancient to Modern time*s (vol. 1, p. 32):Numbers to the Pythagoreans meant whole numbers only. ...Actual fractions... were employed in commerce, but such commercial uses of arithmetic were outside the pale of Greek mathematics proper. Hence, the Pythagoreans were startled and disturbed by the discovery that some ratios -- for example, the ratio of the hypotenuse of an isosceles right triangle to an arm or the ratio of a diagonal to a side of a square -- cannot be expressed by whole numbers. …The discovery of incommensurable ratios is attributed to Hippasus of Metapontum (5th cent. B.C.). The Pythagoreans were supposed to have thrown Hippasus overboard for having produced an element in the universe which denied the Pythagorean doctrine that all phenomena in the universe can be reduced to whole numbers or their ratios."

In short, this bitter denial of irrational numbers for centuries can be attributed to a conviction or belief that all things should be rational, something that may be called Pythagoreanistic rationalism. Only in the last centuries that Georg Cantor and others investigated irrational numbers.

Weierstrass discussed the real numbers' completeness publicly in the lectures he gave at Berlin University in 1865. Weierstrass's construction of irrational numbers used infinite sets of positive rationals with bounded partial sums. In 1872, Kossak publicized this construction. Later, Pincherle in 1883 and Biermann in 1997 further expounded it. Weierstrass insisted on the foundational importance of the property that an infinite bounded set has a cluster point. Further, he added that a continuous function on a closed interval was bounded and attained its bounds. This statement is his invention.

The students of Weierstrass, notably H. A. Schwarz, who was a student in Berlin 1859-1861, and G. Cantor, a student in Berlin 1863-1866, recognized the importance of Weierstrass's ideas and sought to present a more accessible construction of irrational numbers. In 1872, both Cantor and Heine (to whom Schwarz had been and whom Cantor currently was, an assistant at Halle) published constructions of irrational numbers as rational Cauchy sequences.

Referring back to the question posited earlier in this article whether similar debate concerning intuition and logical processes in these modern days continue, regretfully, the answer is affirmative. The underlying reason behind such continuous debate brings this study to the concept of McGilchrist that might shed light on it.

Contribution of Iain McGilchrist' concept

After discussing the historical origin of the irrational number, the contribution of Iain McGilchrist needs attention. As a psychiatrist, his arguments on the Left and Right brain function mean that the left hemisphere, which usually processes in detailed manner any problem (logically), should not predominate the right brain, capturing holistic and spiritual process. McGilchrist might echo the words of Blaise Pascal, a great mathematician from 16th century: "*The heart has its Logic, which reason cannot understand*."

In that sense, the left brain function should and could not rule over the right brain. In other words, in the spiritual realm, especially in worshipping God, the emissary who is the logical process should not dominate the human's heart as its Master. It should be the other way around.

This problem of choosing between Logic or going beyond Logic or rationality to go beyond rational thinking or intuition can be traced back even to the classical history of mathematics. As discussed in the preceding section, Pythagoreans overly worshipped rationality and Logic in mathematics up to the point they could not absorb the shock when one of their disciples found an irrational number. The shock caused Pythagoreans to let the disciple get drown in the sea. In short, the Pythagoreans cannot fathom the contribution of the human brain's right-sphere in pursuing truth.

Similarly, people cannot easily accept several mathematics inventions, such as transcendental numbers, complex numbers, transfinite set, Cantor sets, or non-Diophantine arithmetics.

Philosophy of Logic and Cognitive Linguistic Theory

In 1918, the Austrian philosopher Ludwig Wittgenstein wrote the Tractatus Logico Filosoficus. Its content is to identify the relationship between language and reality and

formulate the boundaries of science. This work emerged because he was concerned about seeing the many languages of philosophy and science collide and confuse people.

In this first work, Wittgenstein makes seven propositions. One of which is:

> *A preposition is a picture of reality: for if I understand a preposition, I know the situation that it represents. And I understand the preposition without having had its sense explained to me. A preposition show its sense. A preposition shows how things stand if it it is true. And says that they do so stand.*[2]

Thus, Wittgenstein stressed that the world is not a cumulation of things but facts. To clarify his proposition, he described the differences between fact, forms, and substance.[3] Further, deviating from Immanuel Kant, for Wittgenstein, the substance only exists in the space of the world. The world consists of interrelated facts. Thus, humans make an effort to map or depict it. Language, whether it is oral, mathematical, artistic, or other kinds of symbols, are a human's effort to make such maps or pictures, but it needs roles as it only serves as a projection of reality or the world.[4]

Wittgenstein also emphasizes that reality is complicated and ever-changing. Therefore, the effort to depict or map it needs more than the rational approach as human Logic can be paradoxical.[5] Thus, mathematical language or symbol only serves essentially as symbols that interact and need structure.

In the second phase of his thought, Wittgenstein realized that all language as the projection of reality exists in societal contexts. In his second work, *Philosophical Investigation,* he formulated a *Language Game Theory*. His work is often multi interpretable. Nevertheless, Wittgenstein often signified that he opened a room of

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[2] Vsevolod Ladov, "Wittgenstein's Tractatus Logico-Philosophicus and a Hierarchical Approach to Solving Logical Paradoxes," *Filosofija, Sociologija* 30, no. 1 (2019): 4.021-4022, doi:10.6001/fil-soc.v30i1.3914.

[3] DAVID MILLER, "The Uniqueness of Atomic Facts in Wittgenstein's Tractatus," *Theoria* 43, no. 3 (1977): 174–85, doi:10.1111/j.1755-2567.1977.tb00786.x.

[4] Antonio Manuel Liz Gutiérrez, "ZALABARDO, José Luis (2015): Represention and Reality in Wittgenstein's Tractatus. Oxford: Oxford University Press.," *Daímon*, no. 75 (2018), doi:10.6018/daimon/346951.

[5] Ladov, "Wittgenstein's Tractatus Logico-Philosophicus and a Hierarchical Approach to Solving Logical Paradoxes."

intuition or irrationality in the process of language creation. The name Language Game indicates that there are rational rules in the game and intuitive ways, and spontaneity. Later, in 1970, a concept emerges with the philosophy of language from Wittgenstein as its backbone.

The spread of the *Cognitive Linguistics theory* shows dynamic energy that contributes to various frameworks for studying a natural language. This theory explores the meaning side of language. Thus, linguistic form and later symbols in their various forms become the focus to delve as the expressions of meaning.[6] According to the framework, meaning is not something that exists in isolation, but it connects and integrates with the full spectrum of human experience—something that Wittgenstein has stated before.

The basic concepts of Cognitive Linguistics encompass conceptual metaphor, image schemas, mental spaces, construction grammar, prototypicality and radial sets. The founding fathers of this theory are George Lakoff and Mark Johnson.[7] Basically, the theory states that there are the concrete domain of a language and an abstract concept that the concrete domain signifies. Whatever aspects one purposely emphasizes or downplays in the concrete form indicate the abstract concepts. Thus, if one states that reality is like a dance, the dance as a concrete experience that most people know means there are aspects of movement, beauty, and artistic sense in that concrete domain. Dance as such will indicate that life also has movement, beauty, and artistic dimension.

The role of Neutrosophic Logic

To summarize, any effort to depict or map life or reality as an abstract substance needs to use real life or concrete experience to arrive at such an understanding. To choose the concrete experience and to connect it with the abstract domain, one needs intuition.

As we emphasize on our paper [8]:

[6] Dirk Geeraerts, *Cognitive Linguistics: Basic Readings*, *Cognitive Linguistics: Basic Readings*, 2008.

[7] G. Lakoff and M. Johnson, "Conceptual Metaphor in Everyday Language," in *Shaping Entrepreneurship Research (Eds, Saras D. Sarasvathy, Nicholas Dew, Sankaran Venkatarama)* (Abingdon on the Thames: Routledge, 2020), 475–504, doi:10.4324/9781315161921-21.

> "More "right brain" activity, based on direct experiences, leads to direct experiences of the Divine. Your "inner vision" (the "mind's eye") can help readers in this, and in many other ways. The inner vision is also the seat of many of the intuitive faculties, which are experiencable facts, not imaginings. That means the information obtained by the intuitive faculty is verifiable and reproducibly observable.
>
> In order to do that, the Balanced Brain is the most efficacious way to function, as well as the most efficient, and the most comfortable.
>
> To obtain the Balanced Brain, the person usually needs to spend a great deal of their spare time being receptive, being the "receiver", being accepting and exploring, and not using the analytical intellect, but instead, spending time in the Now and in the Senses and Sensitivities. This is best enjoyed in Natural settings."

Therefore, if we ask: How can we rectify the problem of overemphasizing rationality in mathematics and beyond? McGilchrist's concept and Conceptual Linguistics theory can shed light. From Neutrosophic Logic viewpoint, this article recommends that a combination of both the intuitive aspect of the right hemisphere and the analytic or logical thinking processes of the left brain is needed to create a holistic approach. That is why we suggested the term: intuilytics, to capture the essence of the Balanced Brain [8].

In other words, if we recall McGilchrist's book theme: the Master (right brain) governs the direction, and then the logical process keeps on finding the detailed answer or paths.

Concluding remarks

The three analyses yield a result that the rational number, symbol, or approach is insufficient by itself. Humans need a space for intuition or irrationality to pursue reality or truth without underestimating rational language contribution in mathematics or other domain of sciences.

So, how can we rectify the problem of overemphasizing rationality in mathematics and beyond? McGilchrist's concept and Conceptual Linguistics theory can shed light. This article recommends that a combination of both the intuitive aspect of the right

hemisphere and the analytic or logical thinking processes of the left brain is needed to create a holistic approach. The term can be: *intuilytics*. In other words, the Master (right brain) governs the direction, and then the logical process keeps on finding the detailed answer or paths.

Returning to the "Man Who Knew Infinity" movie, the lesson learned is as follow: Ramanujan led the discovery of the partition theorem, then he tried to find the proof with his logical processes.

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